

Euler's Method Approximation

What is Euler's method?

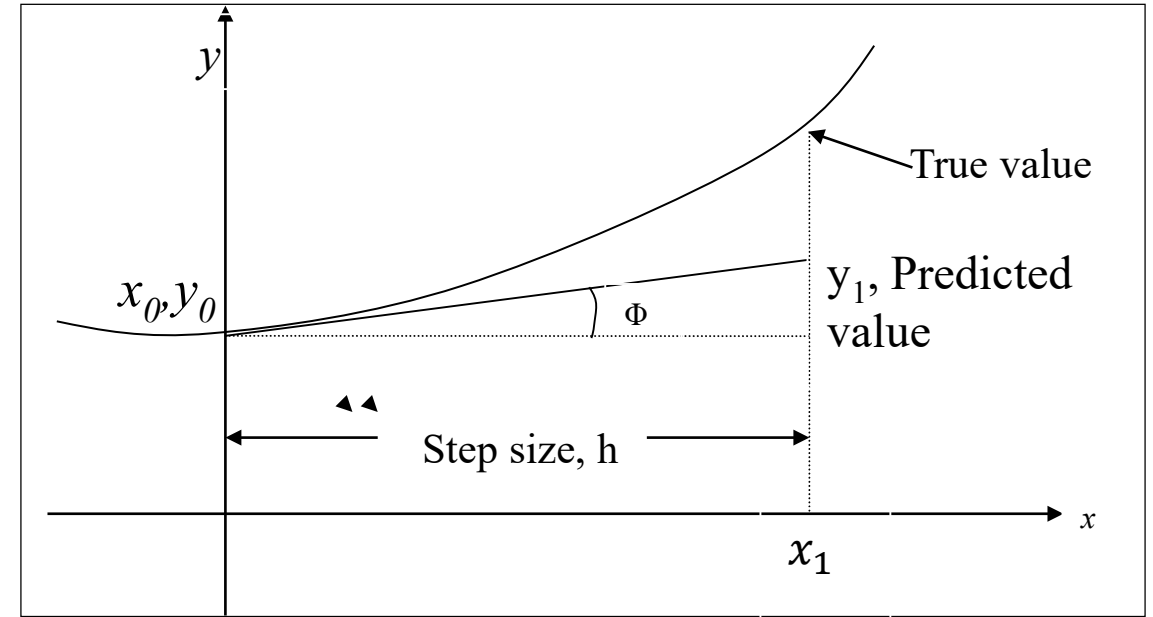
- ✓ Euler's method is a numerical technique to solve ordinary differential equations of the form

$$\frac{dy}{dx} = f(x, y), \quad y(0) = y_0$$

- ✓ So only first order ordinary differential equations can be solved by using Euler's method.
- ✓ How does one write a first order differential equation in the above form?

Derivation of Euler's method

- ✓ At $x = 0$, we are given the value of $y = y_0$
- ✓ Let us call $x = 0$ as x_0 .
- ✓ Now since we know the slope of y with respect to x , that is, $f(x, y)$, then at $x = x_0$, the slope is. $f(x_0, y_0)$
- ✓ Both x_0 and y_0 are known from the initial condition $y(x_0) = y_0$



Graphical interpretation of the first step of Euler's method

Euler's Method

$$\frac{dy}{dx} = f(x, y), \quad y(0) = y_0$$

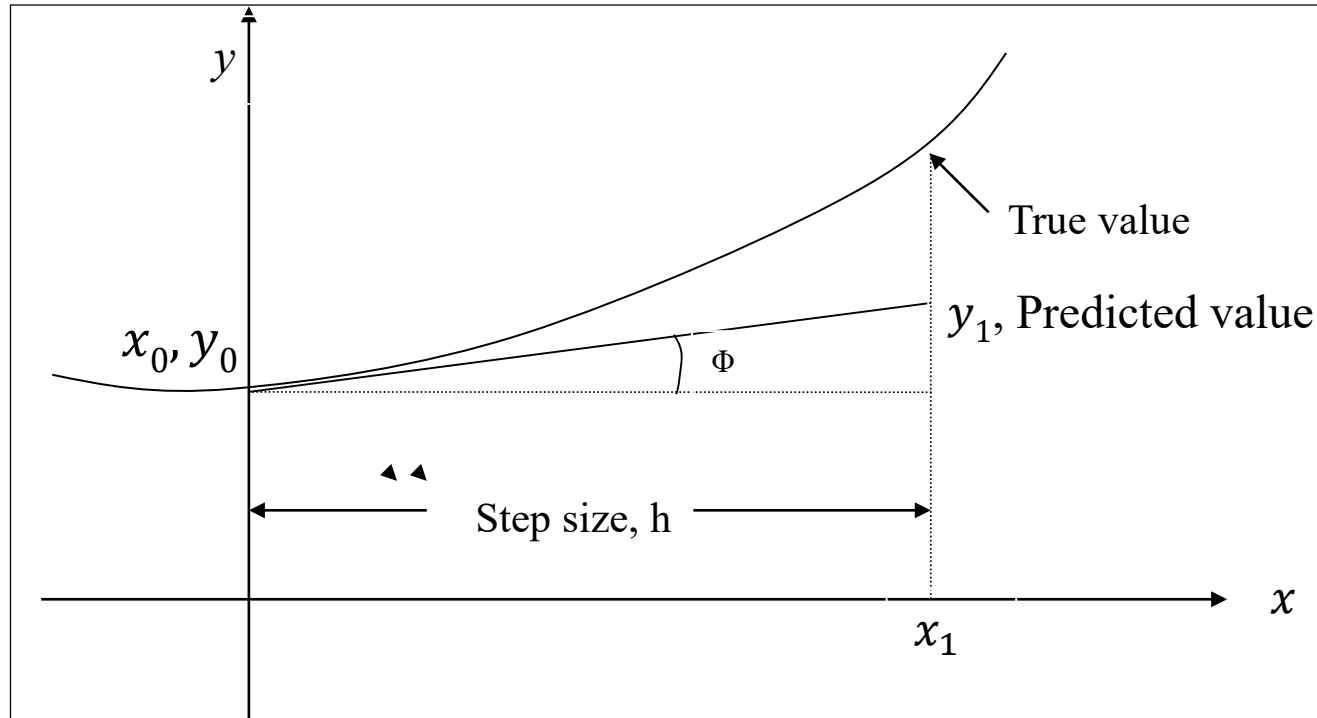
The slope at $x = x_0$ is

$$\begin{aligned} \text{Slope} &= \frac{\text{Rise}}{\text{Run}} \\ &= \frac{y_1 - y_0}{x_1 - x_0} \\ &= f(x_0, y_0) \end{aligned}$$

$$y_1 = y_0 + f(x_0, y_0)(x_1 - x_0)$$

Calling $x_1 - x_0$ the step size h ,
we get

$$y_1 = y_0 + f(x_0, y_0)h$$



Graphical interpretation of the first step of Euler's method

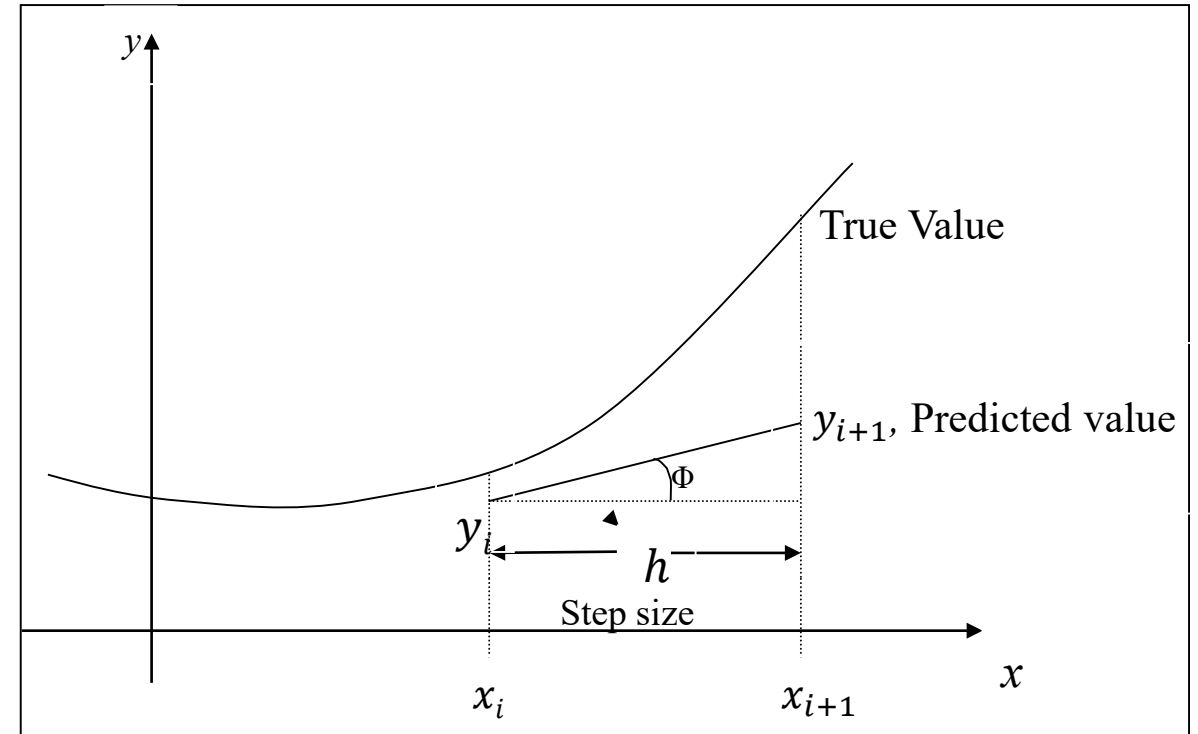
Euler's Method

- ✓ One can now use the value of y_1 (an approximate value of y at $x = x_1$ to calculate y_2 , and that would be the predicted value at x_2
- ✓ given by $y_2 = y_1 + hf(x_1, y_1)$ with $x_2 = x_1 + h$
- ✓ Based on the above equations, if we now know the value of $y = y_i$ at x_i , then

$$y_{i+1} = y_i + hf(x_i, y_i)$$

$$x_{i+1} = x_i + h$$

- ✓ This formula is known as **Euler's method** and is illustrated graphically.



General graphical interpretation of Euler's method